Survivability Model for Voice over Internet Protocol using Markov Regenerative Process

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I. Introduction

Voice over Internet Protocol (VoIP) is a general term for a family of transmission technologies for delivery of voice communications over IP networks such as the Internet or other packet-switched networks. However, it is challenged in the field of quality of service (QoS), like latency, echo, jitter, packet loss, power failure, lack of redundancy etc. The incoming traffic is one of the factors responsible for the system resource degradation and hence it is important to study the traffic behavior so as to manipulate the traffic and make efforts to improve the system.

We assume four stages of degradation. In the first stage the calls operate comfortably. In the second stage, there is some degradation, however the effects are not very disturbing to the user. In the third stage, the effects become quite prominent/visible, however the call is able to survive. But at the fourth stage, survival of calls become impossible and the system would require repair. We develop an analytical model to analyze the survivability of such a system, with increasing number of users, through a Markov Regenerative Process (MRGP). The model is solved for steady state survivability measures. A numerical example is presented to illustrate the applicability of the model. With the help of this model, strategies can be proposed to optimize the available resources, and to regulate the call traffic in a suitable way.

II. Model

Consider a system which accepts atmost n calls. Considering the system degradation into four stages, we partition the optimum number of calls supported at each stage as follows: h calls at stage 2; g calls at stage 3 and the remaining $n-g-h$ at stage 1. Consider a system starting in the 'robust state', stage 1, where all resources are available. As soon as the $(n-g-h)^{th}$ call arrives, there is a degradation in the level of resources and the system reaches stage 2. At this stage a check, say C_1, may be triggered, to examine whether the system resources can support the ongoing and further incoming calls, without affecting the calls quality. If it is not possible to support the ongoing call, then rejuvenation (R) is triggered and the system is restored to the last good state, i.e. the last call at which all resources were available. If sufficient resources are still available to support the ongoing call, then the call continues; else if checking is still going on and more calls pour in (since in a realistic situation we cannot control the incoming of calls) then checking continues. But if upcoming calls cause further degradation of resources, the system moves to stage 3. Here also the system undergoes a similar check say C_2. When the n^{th} call arrives and optimum level is reached, it is assumed that the degradation caused in the resources cannot support the system anymore and the system requires a repair.

This scenario is depicted via a 2-dimensional stochastic

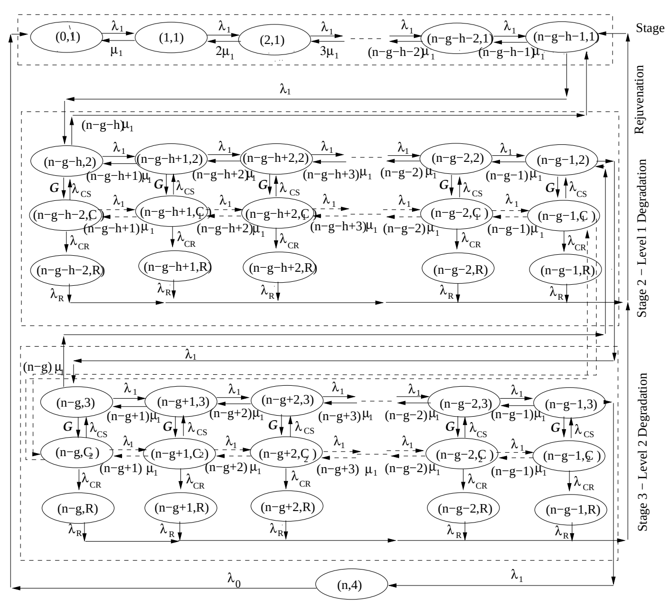

Fig. 1. State Transition Diagram

process: $\{(X(t), Y(t)) : t \geq 0\}$, where $X(t)$ is the number of ongoing calls and $Y(t)$ denotes the level of degradation the system may be in, at time t. The state space of X is $S_X = \{0, 1, 2, \ldots, n\}$ and that of Y is $S_Y = \{1, 2, 3, C_1, C_2, R, 4\}$. The state transition diagram for the same is given in Fig 1.

It is assumed that call inter-arrival and call completion follow independent exponential distributions with rates λ_1 and μ_1, respectively. The time to invoke a check (C_1 or C_2) follows non exponential distribution denoted by G. Note that it is assumed that while checking is going on, a call can arrive, but checking still continues. This is depicted in

the diagram via the transition (shown by a dotted arc in the diagram) (i, C_k) to (j, C_k), $i \geq n-g-h$, $j = i+1, i-1$ and $k = 1, 2$. However if checking results indicate that the system resources are insufficient, then rejuvenation is triggered with an exponentially distributed time with parameter λ_{CR}. Further time for rejuvenation is exponential distribution with parameter λ_R. However, after the check, the system may return to the ongoing call with exponentially distributed time having parameter λ_{CS}. As the n^{th} call arrives, there is a complete failure and repair starts immediately taking an exponentially distributed time with parameter λ_0.
Hence, of all the transitions, only

(i) (i, C_j) to $(i, j+1)$, $i \geq n-g-h,\ j = 1, 2$
(ii) (i, C_j) to (i, R), $i \geq n-g-h,\ j = 1, 2$
(iii) (i, R) to $(n-g-h-1, 1)$
(iv) $(n, 4)$ to $(0, 1)$

are the regeneration time epochs. The system therefore can be modeled using MRGP. Mathematically, if T denotes the set of time epochs of all possible transitions, T_R as the set of all regeneration time epochs, $(T_R \subseteq T)$, $Z_T = \{Z_{t_n} = (X_{t_n}, Y_{t_n}) : t_n \varepsilon T\}$, the state of system at various time epochs or transition time, then $\{(Z_{t_n}, t_n) : t_n \varepsilon T\}$ constitutes a MRGP with $\{(Z_{t_{n_k}}, t_{n_k}) : t_{n_k} \varepsilon T_R, n_k \varepsilon N\}$ and $\{Z_{t_{n_k}} : t_{n_k} \varepsilon T_R, n_k \varepsilon N\}$ being the underlying SMP and DTMC, respectively. We can determine the steady state probabilities of MRGP using the software tool SHARPE [2].

III. Numerical Results and Conclusions

For numerical illustration of the model, we have assumed the following values of the parameters mentioned above: $n = 10$; $g = 2$; $h = 4$, $n - g - h = 4$. The transition rates are taken as : $\lambda_1 = 5$, $\mu_1 = 3$, Hyperexponential distribution $H = \mathcal{HE}(5, 4, 4)$, $\lambda_{CS} = 3$, $\lambda_{CR} = 1.5$, $\lambda_R = 2$, $\lambda_0 = 3$. Evaluating this model in SHARPE we get the following state probabilities:

State	Probability	State	Probability
(0,1)	1.58132775e-001	(5,C_1)	2.40502725e-002
(1,1)	2.63533209e-001	(6,C_1)	6.44917189e-003
(2,1)	2.19600300e-001	(7,C_1)	1.47950361e-003
(3,1)	1.21993029e-001	(8,C_2)	2.96399902e-004
(4,2)	3.75111247e-002	(9,C_2)	5.25335838e-005
(5,2)	9.94427136e-003	(4,R)	5.56377902e-002
(6,2)	2.30038707e-003	(5,R)	1.80377044e-002
(7,2)	4.70062793e-004	(6,R)	4.83687892e-003
(8,3)	8.52745670e-005	(7,R)	1.10962771e-003
(9,3)	1.28489127e-005	(8,R)	2.22299926e-004
(10,4)	2.14148544e-005	(9,R)	3.94001878e-005
(4,C_1)	7.41837202e-002		

- Since $\{(0,1), \ldots, (9,3), (10,4), (4, C_1), \ldots, (9, C_2)\}$ are up states, the system availability is given by: *Availability*=$\pi_{01} + \pi_{11} + \cdots + \pi_{93} + \pi_{4C_1} + \ldots \pi_{9C_2}$
$= P[((0,1), \ldots, (9,3), (10,4), (4, C_1), \ldots, (9, C_2))]$
$= 0.9201$
where π_{ij} is the steady state probability of state (i, j)
- *Call Dropping Probability: (CDP)* Probability that some ongoing calls are dropped. In the model this happens when the system reaches the rejuvenation state.
$= P[(4, R), \ldots, (9, R)] = 0.0799$
- *Call Blocking Probability: (CBP)* The probability of the system, when it cannot accept more new calls. This happens when the system reaches the failure state = 2.14148544e-005.

Further we also see the relation between changing the parameters of Hyperexponential distribution and λ_{CR} which is depicted in the following table.

$\mathcal{HE}(\mu_1, \mu_2, \rho)$	λ_{CR}	Availability	CBP	CDP
$\mathcal{HE}(5,4,4)$	1.5	.9201	2.14148e-005	.0799
$\mathcal{HE}(5,4,4)$	1	.9358	2.69120e-005	.0642
$\mathcal{HE}(5,4,4)$	0.5	.9596	3.54193e-005	.0404
$\mathcal{HE}(4,4,4)$	1.5	.9554	4.16688e-005	.0446
$\mathcal{HE}(4,4,4)$	1	.9652	4.65857e-005	.0348
$\mathcal{HE}(4,4,4)$	0.5	.9790	5.35861e-005	.0209

That is we observe that with the increase in the frequency of rejuvenation, the availability of the system increases, but the call dropping probability also increases. However the system reaches the failure rate less frequently, for any $\mathcal{HE}(\mu_1, \mu_2, \rho)$.

Similarly, we observe from the following table, for $\lambda_{CR} = 1$:

$\mathcal{HE}(\mu_1, \mu_2, \rho)$	λ_R	Availability	CBP	CDP
$\mathcal{HE}(4,4,4)$	3	.9765	4.71318e-005	0.0235
$\mathcal{HE}(4,4,4)$	4	.9823	4.74098e-005	0.0177

As time taken to rejuvenation increases, the system availability increases.
The results can be further used to calculate various survivability measures such as if $(n, 4)$ is an absorbing state, then the system eventually fails with the call arrival and degrading resources. Hence the reliability of the system can be calculated. The measures can also be counter checked via simulation.

References

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[2] Wei Xie, *Markov Regenerative Process in SHARPE*, Master Thesis, Duke University, 1999.